Mathematical analysis on Mullins' model for thermal grooving

Tomoro Asai

(joint work with Yoshikazu Giga)
tasai@ms.u-tokyo.ac.jp
Graduate School of Mathematical Sciences
University of Tokyo, Komaba 3-8-1, Meguro-ku, Tokyo 153-8914, Japan

November 22, 2014

Introduction

- It is well-known that a groove develops on the surface of the material when a grain boundary emerges to intersect the surface.
- A vertical flat grain boundary meets a horizontal flat surface. Immediately, the grain boundary forms a groove in the surface with a certain angle at the triple point.

Thermal grooving

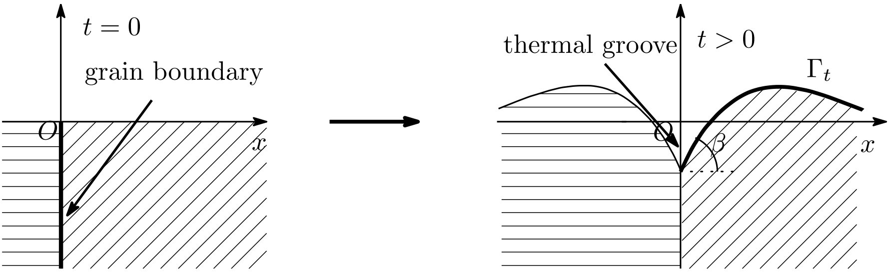

Figure: thermal groove by surface diffusion

Mullins gave two theories to explain this phenomenon: evaporation-condensation and surface diffusion.

The surface diffusion flow with nonlinear boundary conditions

- We introduce the problem which was proposed by Mullins '57. Its precise form is

$$\begin{cases} V = -\partial_s^2 \kappa, & \text{on } \Gamma_t, \\ \sphericalangle(\Gamma_t, y\text{-axis}) = \dfrac{\pi}{2} - \beta, & \\ \partial_s \kappa = 0, & \text{on } y\text{-axis and } \Gamma_t. \end{cases}$$

Self-similar solution

Mullins tried to investigate thermal groove via mathematical analysis on self-similar solution.

Definition (Self-similar solution)

A function $w : [0, \infty) \times [0, \infty) \to \mathbf{R}$ is called a self-similar solution of provided that w is a solution and $w^\lambda(x, t) = w(x, t)$ for all $x > 0, t > 0, \lambda > 0$ $(w^\lambda(x, t) := \lambda^{-1} w(\lambda x, \lambda^4 t))$.

Problem: Does there exist a bounded-in-space self-similar solution?

The surface diffusion flow with linear boundary conditions

In this talk, we show the unique existence of the solution of the surface diffusion flow with linear boundary conditions:

$$(NBP)\begin{cases} \dfrac{\partial v}{\partial t} = -\dfrac{\partial}{\partial x}\left[\dfrac{1}{(1+v_x^2)^{1/2}}\dfrac{\partial}{\partial x}\left(\dfrac{v_{xx}}{(1+v_x^2)^{3/2}}\right)\right], \\ \left.\dfrac{\partial v}{\partial x}\right|_{x=0} = \tan\beta, \\ \left.\dfrac{\partial^3 v}{\partial x^3}\right|_{x=0} = 0, \\ v(x,0) = a(x). \end{cases}$$

Mathematical result (A.-Giga, to appear in *Interfaces and Free Boundaries*)

Theorem (Existence of self-similar solution)

There exists a unique bounded-in-space self-similar solution v to (NBP), which has a smooth profile $Z = w(\cdot, 1)$ provided that β is small.

We set or $\lambda > 0$

$$v_\lambda(x,t) := \frac{1}{\lambda} v(\lambda x, \lambda^4 t), \quad a_\lambda(x) := \frac{1}{\lambda} a(\lambda x)$$

Theorem (Stability of the self-similar solution)

The rescaled v_λ uniformly converges to the self-similar solution w as $\lambda \to \infty$ on any compact sets in $(0, T] \times \mathbf{R}$. In particular, $t^{-1/4} v(t^{1/4} x, t) \to Z(x)$ as $t \to \infty$ uniformly in $[0, \infty)$ when Z is the profile function of w with the small initial data $a(x)$.